

Image recognition with an adiabatic quantum computer

I. Mapping to quadratic unconstrained binary optimization

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Abstract

Many artificial intelligence (AI) problems naturally map to NP-hard optimization problems. This has the interesting consequence that enabling human-level capability in machines often requires systems that can handle formally intractable problems. This issue can sometimes (but possibly not always) be resolved by building special-purpose heuristic algorithms, tailored to the problem in question. Because of the continued difficulties in automating certain tasks that are natural for humans, there remains a strong motivation for AI researchers to investigate and apply new algorithms and techniques to hard AI problems. Recently a novel class of relevant algorithms that require quantum mechanical hardware have been proposed. These algorithms, referred to as quantum adiabatic algorithms, represent a new approach to designing both complete and heuristic solvers for NP-hard optimization problems. In this work we describe how to formulate image recognition, which is a canonical NP-hard AI problem, as a Quadratic Unconstrained Binary Optimization (QUBO) problem. The QUBO format corresponds to the input format required for D-Wave superconducting adiabatic quantum computing (AQC) processors.

1 Introduction

Humans currently have substantial performance advantages over machines in several areas, including object recognition, knowledge representation, reasoning, learning and natural language processing [RN03]. Intriguingly, most of the hard problems arising in these areas can naturally be cast as NP-hard optimization problems, with the majority reducible to pattern matching problems such as maximum common subgraph [Smi99, EV07, Bun00, BDK⁺08, Sin02]. The formal intractability of most problems associated with human intelligence is at the heart of the continued difficulties AI researchers face in mimicking or surpassing human capabilities in these areas.

It may seem surprising that capabilities that we take for granted and perform quite easily could be computationally intractable. However it is important to remember that this intractability does not preclude efficient generation of approximate solutions. In practice, exact solutions to optimization problems arising in AI are not required. Generally there is a graceful degradation of performance as a solution moves away from global optimality. Because of this behavior the ideal computational

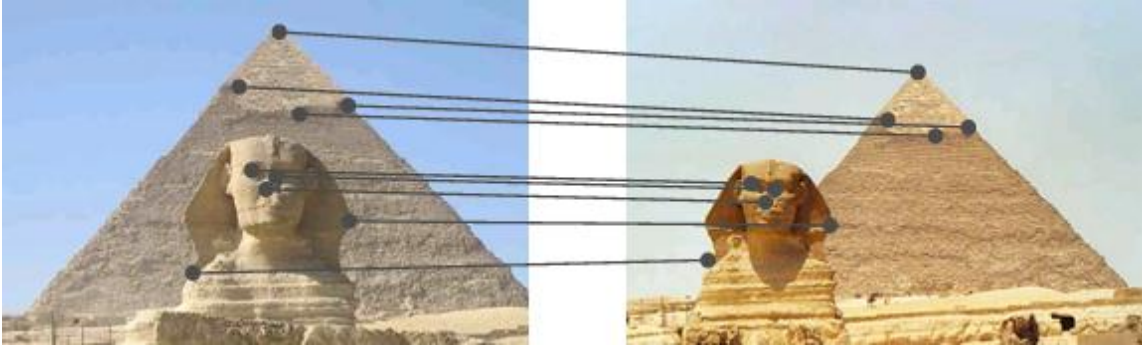


Figure 1: Object recognition by image matching proceeds by pairing points in two images that correspond to the same structure in the outside world. In the algorithms considered here, both feature similarity and geometric consistency are considered in determining to what extent two images are similar.

approach is to use specialized heuristic algorithms to attack these problems [Sim95]. It is interesting to note that human brains are thought to contain structures specialized for pattern matching (‘wetware heuristics’) that are used to support a variety of capabilities for which humans still hold a performance advantage over machines, and that these structures have been used as inspirations for development of successful heuristic algorithms [Sin02, Mou97, Mac91].

In this article we focus on the quintessential pattern recognition problem of deciding whether two images contain the same object. This is a typical example of a capability in which humans outperform modern computing systems and can be thought of as an NP-hard optimization problem. We begin to explore whether quantum adiabatic algorithms [EFS00, CFGG00, BBTA99, SMTC02] can be employed to obtain better solutions to this problem than can be achieved with classical optimization algorithms. The first step in this exploration is to map image recognition into the particular input format required for running quantum adiabatic algorithms on D-Wave superconducting AQC processors.

2 Image matching

A popular method to determine whether two images contain the same object is *image matching*. Image matching in its simplest form attempts to find pairs of image features from two images that correspond to the same physical structure. An image feature is a vector that describes the neighborhood of a given image location. In order to find corresponding features two factors are typically considered: *feature similarity*, as for instance determined by the scalar product between feature vectors, and *geometric consistency*. The latter is best defined when looking at rigid objects. In this case the feature displacements are not random but exhibit correlations brought about by a change in viewpoint. For instance, if the camera moves to the left we observe translations of the feature locations in the image to the right. If the object is deformable or articulate then the feature displacements are not solely determined by the camera viewpoint anymore but one can



Figure 2: Representation of images as labeled graphs. Shown are three exemplary interest points for each image. The number of interest points detected is content dependent but is on the order of several hundred for 640x480 resolution images with content as shown. Each interest point is assigned a position, scale, and orientation [Low99]. In the figure the scale is indicated by a circle and the orientation by a pointer. This information can be used to characterize the relative pose and position of two interest points denoted by the vectors \vec{g} next to the dotted lines.

still expect that neighboring features tend to move in a similar way. Thus image matching can be cast as an optimization problem in which one attempts to minimize an objective function that consists of two terms. The first term penalizes mismatches between features drawn from image one and placed at corresponding locations in image two. The second term enforces spatial consistency between neighboring matches by measuring the divergence between them. It has been shown that this constitutes an NP-hard optimization problem [FH05].

3 Mapping Image Matching to a Quadratic Optimization Problem

D-Wave AQC processors take as input problems of the form

$$\vec{x}_{opt} = \arg \min \left\{ \sum_{i \leq j=1}^N Q_{ij} x_i x_j \right\}, x_i \in \{0, 1\} \quad (1)$$

which are typically referred to as Quadratic Unconstrained Binary Optimization (QUBO) problems. Physicists will recognize this objective as being closely related to the Ising energy function. In order to use D-Wave hardware to run quantum adiabatic algorithms, the problem of interest must first be converted to this format. In this section we will describe how to perform this conversion for image matching.

3.1 Representing an image as a labeled graph

As a first step towards casting image matching problems as QUBO minimization, it is convenient to reduce the amount of data and to focus on features that are sufficiently unique and robust under

small image transformations that finding a correspondence is well defined. This reduction to salient image structures is accomplished with an interest point operator. Many versions of interest point operators have been described in the literature [MS04, MTS⁺05]. In our implementation we use a Laplacian of Gaussians operator. Each interest point i is described by a normalized feature vector \vec{f}_i called a *local descriptor*. We employ a new local descriptor called CONGAS that is based on Gabor wavelets of varying scale and orientation drawn from a space variant grid around an interest point [BN08][WFKvdM97]. Each pair of interest points is associated with a quantity \vec{g}_{ij} that describes the geometric relationship between the interest points. \vec{g}_{ij} measures the translation between interest points as well as the difference in local scale and orientation. The quantity \vec{g}_{ij} is normalized for global translation, rotation and scaling. Using these concepts, any image can be represented by a labeled graph G (see Fig. 2).

3.2 Generating a conflict graph G_C from two image graphs G_1 and G_2

Images I_1 and I_2 give rise to collections of interest points. We associate the M interest points of I_1 with vertices in a labeled graph G_1 having M nodes. The i^{th} vertex in G_1 is labeled with the feature vector \vec{f}_i . Edges in G_1 between feature points i and j are labeled with \vec{g}_{ij} . Edges characterize the geometric relationships between feature vectors. Similarly, image I_2 is represented as a labeled graph G_2 over N features (vertices). In this representation the similarity of two images is specified by the similarity of the two labeled graphs.

For pairs of interest points drawn from each image ($i \in G_1, \alpha \in G_2$) we calculate the similarity of the associated feature vectors $d(i, \alpha) = d_{feat}(\vec{f}_i, \vec{f}_\alpha)$. Note that when using subscripts we will use the convention that Latin subscripts refer to G_1 and Greek subscripts to G_2 . $d(i, \alpha)$ is a measure of the similarity of the interest points i and α . A common choice to measure the distance between two feature vectors, which we assume to be normalized such that $|d(i, \alpha)| \leq 1$, is the scalar product. Of all of the potential matches between interest points in G_1 and G_2 , some will be excellent (corresponding to $d(i, \alpha)$ close to one) and some will be poor. As we do not want to keep poor matches, we introduce a point-wise inclusion threshold T_{feat} . We define a pair of points (i, α) to be a potential match if $d(i, \alpha) > T_{feat}$. Increasing T_{feat} decreases the number of potential matches i.e. it increases the standard for what constitutes an acceptable point-wise match.

3.3 Generation of vertices in the conflict graph G_C

We generate a conflict graph G_C from G_1 and G_2 to measure the similarity of the two graphical representations of the images. Starting from the largest $d(i, \alpha)$ we add a vertex $V_{i\alpha}$ to G_C until either all potential matches have been included or the number of vertices reaches a hardware dependent limit L . Vertex $V_{i\alpha}$ corresponds to an association of feature i in G_1 with feature α in G_2 .

3.4 Generation of edges in G_C

Edges $(i, \alpha; j, \beta)$ in G_C encode geometric consistency between feature vectors \vec{f}_i and \vec{f}_j in G_1 and feature vectors \vec{f}_α and \vec{f}_β in G_2 . For all vertex pairs $(V_{i\alpha}, V_{j\beta})$ in G_C where $i \neq j$ and $\alpha \neq \beta$ we calculate $d(i, \alpha, j, \beta) = d_{geom}(\vec{g}_{ij}, \vec{g}_{\alpha\beta})$. This quantity we call the *geometric consistency* of the two pairs of interest points, and is normalized such that $|d(i, \alpha, j, \beta)| \leq 1$. It measures the geometric compatibility of the match pairs (i, α) and (j, β) as the residual differences in local displacement, scale and rotation assignment of the associated interest points after changes due to global translation,

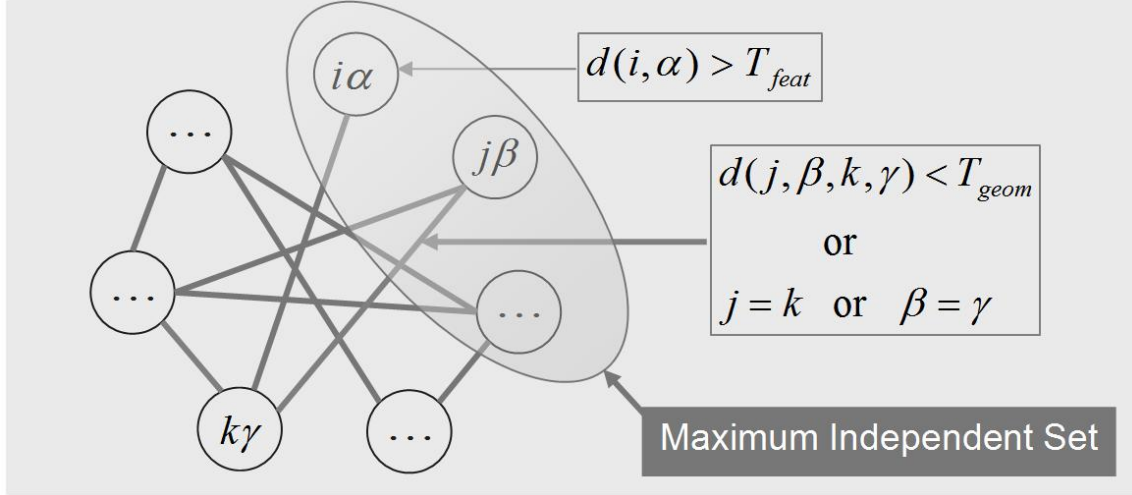


Figure 3: Image matching via determining the maximum independent set (MIS) of a conflict graph. The MIS problem is a special case of the more general QUBO problem class. In the MIS formulation each vertex $V_{i\alpha}$ in the graph represents a match candidate for which the feature similarity exceeded a threshold T_{feat} . An edge is placed between any two matches that are geometrically inconsistent. The MIS represents the largest set of matches that are geometrically consistent.

rotation and scaling have been normalized out. A pair (i, j) and (α, β) are not allowed to match if they are in geometric conflict i.e. if the residual differences are too large. This requirement is enforced by choosing a threshold T_{geom} whereby if $d(i, \alpha, j, \beta) < T_{geom}$ the pair (i, j) and (α, β) are considered to be in geometric conflict. Thus, the prescription for edge drawing in G_C is as follows:

- For all pairs of vertices $(V_{i\alpha}, V_{j\beta})$ in G_C , draw an edge between the vertex pair $(V_{i\alpha}, V_{j\beta})$ if $i = j$ or $\alpha = \beta$
- For all pairs of vertices $(V_{i\alpha}, V_{j\beta})$ in G_C , draw an edge between the vertex pair $(V_{i\alpha}, V_{j\beta})$ if $i \neq j$ and $\alpha \neq \beta$ and $(V_{i\alpha}, V_{j\beta})$ are in geometric conflict (ie. $d(i, \alpha, j, \beta) < T_{geom}$)

In either case the presence of an edge records a conflict in associating i with α and j with β . Note that the first condition ensures that two features which are separate in one image are not be mapped onto a single feature in a second image.

3.5 Structure of G_C

The graph generated by the preceding prescription we call a *conflict graph*. It has by construction at most L vertices, and has arbitrary connectivity with at most $L(L - 1)/2$ edges. The maximum independent set (MIS) of the conflict graph is equivalent to the maximum common subgraph of unlabeled graphs G_1 and G_2 . The MIS provides both a similarity measure (the larger the MIS, the greater the region of mutual overlap) and the largest conflict-free mapping of features in G_1 to features in G_2 . For example if the MIS of G_C is the set $\{V_{i\alpha}, V_{j\beta}, V_{k\gamma}\}$ then the point-wise matches

between the two graphs are $i \leftrightarrow \alpha$, $j \leftrightarrow \beta$, and $k \leftrightarrow \gamma$. Finding the MIS for the conflict graph G_C can be cast as a QUBO by setting $Q_{i\alpha,i\alpha} = -1$ for all vertices and $Q_{i\alpha,j\beta} = L$ whenever there is an edge between (i, α) and (j, β) . The minimum energy configuration enforces $x_{i\alpha} = 1$ if and only if $V_{i\alpha} \in MIS$ and $x_{i\alpha} = 0$ otherwise. More elaborate objectives defining the correspondence between images can be chosen. Those would not necessarily constitute an MIS problem but could still be formulated as a QUBO.

4 Summary and Next Steps

This article presents a description of how to map image recognition problems into the input format required for using D-Wave superconducting AQC processors. This represents a first step in determining to what extent quantum adiabatic algorithms may be useful as components of novel solvers for the NP-hard optimization problems underlying image recognition.

Quantum adiabatic algorithms are to date largely theoretical and verification of their utility as components of either complete or heuristic solvers, and their actual performance in either regime, awaits experimental verification [Llo08]. The next step in our reporting of this work will be a detailed description of the results of solving QUBO instances generated by image matching problems using D-Wave superconducting AQC hardware.

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